Volume 30 Issue 3 Summer 2025

Routines to Scaffold Mathematics Discourse

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About the Author

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Introduction

Facilitating mathematics discourse is a hallmark of reform-oriented mathematics teaching. When mathematics discourse is skillfully facilitated, it can support students to make deeper and sustained meaning of the mathematical concepts they are learning. This is why the National Council for Teachers of Mathematics (NCTM, 2014) identifies Facilitate Meaningful Mathematical Discourse as one of eight effective mathematics teaching practices. It is not easy for instructors to learn to facilitate meaningful mathematics discourse, nor is it ordinary for basic adult education (ABE) students to be familiar with such classroom instructional methods. This article is the first installment of a three-part series about mathematics discourse in ABE classrooms. In this article, we share how the Adult Numeracy in the Digital Era (ANDE) project utilized three signature routines to scaffold teachers' use of and students' participation in mathematics discourse. These three signature routines include: (a) Which one doesn't belong? (b) What do you notice? What do you wonder?, and (c) Comparing solved problems. We share how we purposefully integrated these routines into instruction so that other ABE instructors can similarly do so.

The ANDE project team (see Perry et al., 2025) set out to design a digitally enhanced numeracy curriculum that engages ABE students in rich, context-based mathematics tasks while supporting students' goals of obtaining their general education diploma. Each ANDE lesson is centered on a collaborative activity that requires teachers and students to engage in rich and meaningful mathematics discourse. The ANDE materials, in coordination with instructor professional development, aims to create discourse communities (Hufferd-Ackles et al., 2004) that can be characterized by students:

- explaining to their peers and the teacher their strategies and/or thinking,
- asking one another questions about their work and/or thinking, and
- taking responsibility to monitor their own and co-evaluate their peers' ideas and/or thinking.

Such classrooms are remarkably different from traditional mathematics classrooms that requires students to focus on memorizing mathematical procedures. Learning to participate in mathematics discourse in this way takes time, practice, and patience. In our initial field tests of the ANDE implementation, we noticed instructors and students were uncomfortable with these new ways of interacting. As the curriculum and professional development was revised, we developed signature routines as a way to welcome instructors and students into new ways of being while teaching and learning mathematics. We noticed that over time students felt more comfortable participating in the routines. The new ways of interacting cultivated during the signature routines, started to be taken up naturally at other times in the lessons.

We have incorporated one signature routine into the beginning of each lesson, (e.g., a warm-up activity) and the content of the routine is purposefully selected to support the lesson's mathematics learning goals. Sometimes a specific image and/or representation that appears later in the lesson is used during the signature routine. They are all open prompts which encourage discussion rather than focusing on getting the answer. Rarely does a signature routine have a single right answer. It is important to keep the routine friendly and non-judgmental because this helps to maximize participation. Rather than correcting the students' thinking, it is more important for the teacher to uncover why a student is thinking/reasoning the way they are. In this way, the routines support instructors to formatively assess the knowledge, skills, and perspectives that students are bringing to the lesson. This can help the instructor decide what support students need during a lesson and what concepts may need more emphasis while summarizing the lesson. The instructor can use this information later in the lesson to steer students toward conventional thinking.

Signature Routines Support Teachers' Learning

We found that the use of the signature routines supported teachers to facilitate mathematical discourse outside of the routines. The signature routines acted like a container for teachers to try out new discourse moves. While the Five Practices of Orchestrating Productive Mathematics Discussion (Smith & Stein, 2011) provides a useful architecture and set of methods for planning and approaching whole class discussions, it requires substantial shifts to practice. The signature routines provide a moment in each class meeting to approximate (Grossman et al., 2009) facilitating mathematics discourse in a contained portion of class. Teachers reflected on their experiences from the signature routines by noticing the discourse moves they tended to rely on, recognizing that some students had more voice in whole-class discussions, and preparing new strategies to open-up the discourse for more students. Practicing facilitating discourse within the container of signature routines prepared them to facilitate more robust mathematical discussions, such as those represented in the Five Practices. As teachers became more adept at facilitating these dialogues, they naturally incorporated these skills into broader class discussions, enriching the overall learning experience for students. The student-centered discourse patterns that begin happening during the signature routine naturally extended into the other learning activities.

We invite you to reflect on these signature routines and transform the mathematical discourse in your adult education classroom. Next, we provide more detail on the three signature routines, and how they can be implemented in the classroom.

Signature Routines in Practice

The purpose of integrating signature routines into the beginning of each lesson was to scaffold teachers' and students' use of new discourse practices. To support teachers, we deliberately focused on three types of discourse moves (Herbel-Eisenmann et al., 2013): Inviting student participation was used for a variety of purposes including initiating discussion, eliciting multiple perspectives, and encouraging students to listen to other students' responses. Orienting to each other's reasoning can allow students to "try on" another way of thinking, encourage students to extend or build upon someone else's approach, and encourage active listening; Asking students to revoice also encourages active listening, check for student understanding, and increase student participation in discourse. In the descriptions provided below, we illustrate only one category of discourse move (i.e., inviting, orienting, and asking) for each signature routine. We encouraged instructors to incorporate one category of discourse move with each signature routine as they started to enact the routines; however, all of the discourse moves could support any of the routines or mixed together.

Which One Doesn't Belong

In the Which One Doesn't Belong (WODB) routine (Andrews & Bandemer, 2025; Danielson, 2023), students are presented with four objects (e.g., four numbers, four different representations, four images) and are asked to identify which one of the four objects doesn't belong in the set, and why. The key to the WODB is that students provide a rationale for why they have chosen any particular object. The set of objects are carefully curated so that it is possible to develop a reason that any one of the objects doesn't belong (while the other three objects all have similar characteristics). To enact this routine, reveal a slide with the four objects to the class, then ask, "Which one doesn't belong? Why?" or "Can you find a reason that one of these doesn't belong in the set?" (see Figure 1, below, for an example). It is important to give students plenty of thinking time. Instructors typically ask their students not to raise their hands once they have an answer.

Which One Doesn't Belong?

$\frac{1}{2}$	$\frac{1}{4}$
$\frac{8}{9}$	$\frac{3}{4}$

Figure 1: Example Which One Doesn't Belong Routine

Teacher notes:

½	• because it is the only fraction that is directly between 0 and 1 on a number line. • because it is the only fraction that when you add it to itself (e.g., double or multiply by two) that equals 1. For example, ½ + ½ = 1, but ¾ + ¾ is 6/4, or 1 and 1/2. • because it is the only fraction with a 2.
¼	• because it is the only fraction that is not 1/n-th away from 1. For instance, ½ + **½** = 1, 8/9 + **1/9** = 1, and ¾ + ¼ = 1. In contrast ¼ + ¼ does not equal 1. • because it is the only fraction where the numerator and denominator are not consecutive integers. • because it is the only fraction that is less than a half.
8/9	• because it is the only fraction with an even numerator. • because it is the only fraction that we don't commonly use the decimal form (students are likely to recognize ½ = 0.5, ¼ = 0.25, and ¾ = 0.75) • because it is the only fraction with and 8 (or a 9). • because it doesn't make a pattern with ¼, ½, and ¾
3/4	• because it is the only fraction with a 3 in it. • because it is the only multiple of ¼ (e.g., ¼ * 3 = ¾)

The example in Figure 1 shows how this routine was utilized to open a lesson on ordering fractions. As you consider the four fractions in Figure 1, which one would you say doesn't belong? Can you develop a reason that any of the other three objects do not belong to the set?

WODB provides an opportunity to formatively assess the attributes of mathematical objects. For instance, when ANDE instructors used Figure 1 they gained insights into students' familiarity with vocabulary such as numerator and denominator, recognition of benchmark fractions, familiarity with unit fractions (e.g., 1/n), and other topics relative to ordered fractions.

When you use the WODB routine with your students, we encourage you to use the following discourse moves to encourage students to share their reasoning. These discourse moves all support instructors to invite students into the conversation.

- Who would like to share their reason for why one of them doesn't belong?
- Did anybody else say another one didn't belong? What was your reason?
- Did anybody choose the same answer, but had a different reason/justification?
- Nobody said that [blank] didn't belong. Can anybody find a reason why you could say this [blank] doesn't belong?

What do you notice? What do you wonder?

When using the What do you notice? What do you wonder? (Notice/Wonder) routine (Ray, 2013), students are presented with a single object (e.g., table, graph) and asked: What do you notice? What do you wonder? The ANDE curricular resources use this to activate students' thinking related to a representation that appears later in the lesson. This routine can also be used as a probe for problem-solving strategies not related to math. Students could look at a coordinate plane, double number line tape diagram, then asked to take note of the important features. Students could first describe what they notice, then ask any questions they have. These wonderings can be questions they could ask (and possibly answer) or questions they have about the representation. To initiate this routine, instructors reveal the representation to the class and ask, "What do you notice?" After ample wait-time, the instructor encourages students to share their noticing then publicly records these responses on chart paper, for example. No judgement should be given to the responses. Then the instructor does the same after asking, "What do you wonder?" During this routine, the teacher doesn't need to respond to students' wonderings. At the end of class, the teacher can return to these wonderings to see which have been resolved and which still require some explanation by the teacher.

The example in Figure 2 shows how we used the Notice/Wonder routine in a lesson about the Cartesian Coordinate plane. In this routine, instructors gleaned insights into students' familiarity with the plane by listening for language related to the x-axis, y-axis, and origin, while also learning how students described locations. Answers may vary from distance between coordinate locations to relative location. This allowed some instructors to differentiate instruction and provide some students with direct instruction related to reading a coordinate plane. We have found the Notice/Wondering routine provides opportunities to formatively assess students' familiarity and fluency with various mathematical representations.

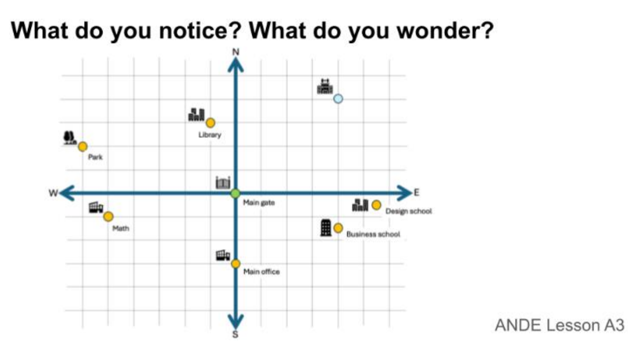


Figure 2: Example Notice/Wonder Routine

Teacher notes:

Students may **notice** several aspects of this representation, such as:	• Each dot is labeled with a different title. • Design school is farthest to the right • The park is farthest to the left • Something unknown is the highest • The Main Office is the lowest
Some additional **noticings** that might support the lesson include:	• The horizontal axis is labeled W and E for West and East. In math this is typically titled "x." • The vertical axis is labeled with N and S for North and South. This is typically titled "y-axis." • The two axis cross at the Main Gate. This would be labeled (0,0) typically. • The axes cut the graph into four areas.
Students will likely have several **wonderings**, such as:	• What do these points mean? • Which two dots are the farthest apart?
Some **wonderings** that might support the lesson include:	• What's the building in Quadrant I (top left)? Why does it have no label? • What is a mathematical way we could label the space between different buildings?

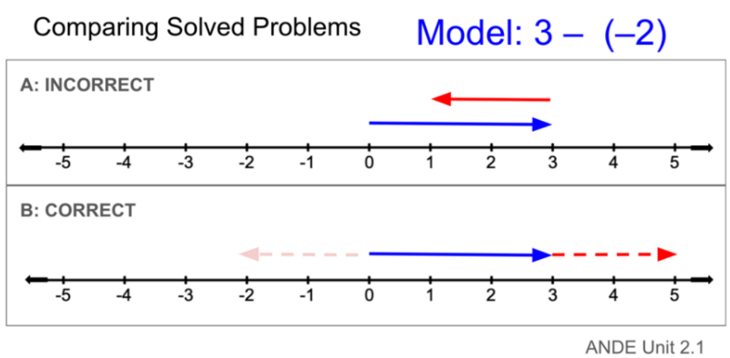
While using the Notice/Wonder routine with your students, we encourage using the following discourse moves designed to encourage students to orient themselves with each others' reasoning.

- Did anybody else notice something similar but stated it a different way?
- Did anybody else ask a similar question but stated it differently?
- We haven't heard from [blank] yet. What did you notice or wonder? Are any of these ideas related to one of yours?

Comparing Solved Problems

The Comparing Solved Problems routine presents students with two worked solutions for the same problem or prompt (Star et al., 2015). In the ANDE materials, students are presented with two computations to compare, and at other times, students are presented with two representations to compare. In all but one of the Comparing Solved Problems, students will see one correct and one incorrect example. The two examples are always clearly marked as either correct or incorrect. In one instance, students compare two strategies with correct solutions. It is critical that students always see a label identifying which solution is correct and which one is incorrect. The purpose of this activity is for students to gain familiarity with why the correct example is correct and identify the error in the incorrect example. As students engage in this activity, they will gain fluency with a computation device or use of a mathematical representation. Students are encouraged to think aloud about what they are seeing, in order to verbalize their reasoning. To enact this routine, the instructor reveals the prompt or problem and the two solutions, then allows for at least 30 seconds for students to think independently to consider how the two solutions differ. Sometimes instructors ask students to share their thinking with a partner before facilitating a whole-class discussion, while at other times they move directly to a whole-class discussion. The discussion opens by the teacher inviting students to share their ideas with a question like: What do you notice is the same or different about the two solution methods shown? The whole class discussion should focus on identifying why–procedurally and conceptually–each solution is correct or incorrect.

The example in Figure 3 shows how this routine was incorporated into a lesson on adding and subtracting integers with a number line representation. As students discuss the example they note that the minuend always extends from zero to its place on the number line. Students compare the placement of the red bar and notice that the incorrect diagram represents 3 – 2. This helps to build understanding about why the bar representing negative two (–2) switches directionality in the diagram so that 3 – (–2) = 3 + 2. The Comparing Solved Problems routine allows students to share and discuss how they are making sense of important mathematical ideas to build stronger understanding of the mathematical concepts and procedures. This routine continues to foster a classroom environment that values exploration and discourse.



Teacher notes:

Figure 3: Example Compare Solved Problems Routine

Students may **notice** several aspects of this representation, such as:	• Each dot is labeled with a different title. • Design school is farthest to the right • The park is farthest to the left • Something unknown is the highest • The Main Office is the lowest
Some additional **noticings** that might support the lesson include:	• The horizontal axis is labeled W and E for West and East. In math this is typically titled "x." • The vertical axis is labeled with N and S for North and South. This is typically titled "y-axis." • The two axis cross at the Main Gate. This would be labeled (0,0) typically. • The axes cut the graph into four areas.

Students will likely have several **wonderings**, such as:	• What do these points mean? • Which two dots are the farthest apart?
Some **wonderings** that might support the lesson include:	• What's the building in Quadrant I (top left)? Why does it have no label? • What is a mathematical way we could label the space between different buildings?

When you use the Comparing Solved Problems routine with your students, we encourage you to use the following discourse moves that encourage students to listen carefully to their peers and build on their ideas by asking students to revoice ideas.

- Can somebody say what [name] said in their own words?
- Who can summarize the questions [or different ideas] that have been shared so far?
- Can you say that again, but without using the word "it" [or another word to encourage precision]?

Conclusion

Mathematics discourse can support students' understanding of mathematical ideas while also providing students with a new sense of what it means to do mathematics, thereby building stronger mathematical identities. In this article, we shared three signature routines that we purposefully integrated in the ANDE curriculum to scaffold both teachers' and students' engagement in mathematics discourse. We hope that you will try out using these routines and discourse moves to welcome your students into new ways of interacting with mathematics and understanding mathematical ideas. In the next issue of The Math Practitioner, we will present additional lessons learned in the ANDE project as well as tools and resources to investigate and bolster mathematics discourse in your ABE classroom.

Acknowledgments

This article was prepared under Grant R305N210029, with full funding from the U.S. Department of Education, Institute of Education Sciences (IES), as indicated in the grant abstract for the project. The content does not necessarily reflect the views or policies of IES or the U.S. Department of Education, nor does mention of trade names, commercial products, or organizations imply endorsement by the U.S. Government.

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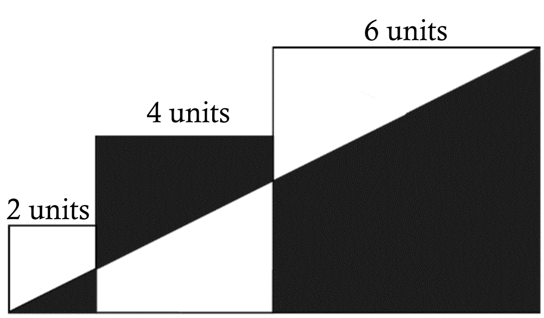
Puzzle Challenge

A Little Math Fun for You



Given three squares with side lengths of 2 units, 4 units, and 6 units respectively as shown. Determine the shaded area.

Ever scroll past a puzzle in your social media feed and *just* have to stop and solve it? That happened to me with this one—I couldn't let it go until I tried it!

Think you can solve it? Show off your skills by sending your answer *and* your work here.

Do you have a puzzle of your own that's too good to keep to yourself? Share it here—you might see it in an upcoming issue!