Promoting Student Learning and Productive Persistence in Developmental Mathematics: Research Frameworks Informing the Carnegie Pathways

Ann R. Edwards and Rachel L. Beattie, Carnegie Foundation for the Advancement of Teaching

Abstract
This paper focuses on two research-based frameworks that inform the design of instruction and promote student success in accelerated, developmental mathematics pathways. These are Learning Opportunities—productive struggle on challenging and relevant tasks, deliberate practice, and explicit connections, and Productive Persistence—promoting students' academic and social mindsets, and good strategies. These frameworks are the foundations of the highly successful Carnegie Pathways (Statway and Quantway), two distinct pathways that take students who place into developmental mathematics through college-level mathematics in one year. In this paper, we describe these research-based frameworks and discuss examples of high impact practices derived from them.

In 2009, the Carnegie Foundation for the Advancement of Teaching engaged a network of practitioners, researchers, designers/developers, and institutional leaders to design and implement two pathways that aim to accelerate community college students' progress through their entire developmental mathematics sequence and a college-level course for credit in a single year—Statway® and Quantway®. Statway integrates developmental mathematics and college-level statistics. Quantway covers developmental mathematics and college-level quantitative reasoning. The Pathways have been remarkably successful, helping thousands of students achieve success in college-level mathematics in a single year and tripling the success rate for college credit completion for students who place into developmental mathematics in half the time (Sowers & Yamada, 2015).
Central to the effectiveness of the Pathways is instruction that incorporates two key catalysts for powerful student learning: (1) the Learning Opportunities—productive struggle, deliberate practice, and explicit connections; and (2) Productive Persistence - promoting students' tenacity and good strategies.
In this paper, we discuss these research-based frameworks and how they inform instruction designed to promote student learning, engagement, and persistence in developmental mathematics. We provide examples of high impact practices derived from these frameworks used by the Pathways network.

The Pathways Learning Opportunities

The National Research Council in *How People Learn* (2005) determined that there are three basic principles of learning: 1) New understandings are constructed on a foundation of existing or prior understandings; 2) The brain forms cognitive schema or networks that are important to emphasize in the learning process; and 3) The ability to self-monitor or possess skills of metacognition enhance learning. The Pathways instructional system addresses the essence of these fundamental principles with "learning opportunities" derived from key research findings in the learning sciences, psychology, and cognitive science that inform the design of Pathways curriculum and instruction—productive struggle on challenging and relevant tasks, explicit connections to concepts, and deliberate practice.

Productive Struggle

Derived in part from research on mathematical sense-making and the development of robust conceptual understandings in mathematics, productive struggle refers to opportunities for students to grapple with important mathematical ideas.

> We use the word struggle to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent. We do not use struggle to mean needless frustration or extreme levels of challenge created by nonsensical or overly difficult problems…The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed. (Hiebert & Grouws, 2007, pp. 387–388)

The ultimate goal of productive struggle is to encourage students to make meaning of mathematical content for themselves. In Pathways instruction, productive struggle most often occurs in collaborative learning settings in which students explore rich mathematical tasks as they develop strategies to investigate the problem situation or question. Students who are productively struggling are engaged and inquiring, repeatedly making guesses and judgments about how to use mathematics to approach the given situation. Promoting productive struggle involves posing tasks that require substantive mathematical thinking and giving students both the time and encouragement within the classroom culture to engage with the problem.

Explicit Connections
By explicit attention to connections, we mean that connections among mathematical facts, procedures, and ideas should be addressed explicitly.

> This could include discussing the mathematical meaning underlying procedures, asking questions about how different solution strategies are similar to and different from each other, considering the ways in which mathematical problems build on each other or are special (or general) cases of each other, attending to the relationships among mathematical ideas, and reminding students about the main point of the lesson and how this point fits within the current sequence of lessons and ideas. (Hiebert & Grouws, 2007, p. 383)

A review of findings from across multiple studies—some teacher-centered, others student-centered—suggest that teaching for conceptual understanding leads to improvement not only in conceptual understanding but also in procedural skill. The reverse has not been found to be true (Hiebert & Grouws, 2007).
Thus, when we suggest that the focus of Pathways instruction is on concepts, we are not suggesting that knowledge of procedures is unimportant, but rather that instruction focused on concepts is the better way to achieve both learning outcomes.

Deliberate Practice
The literature suggests that repeating a behavior over and over is not an effective method of reaching maximal levels of performance. Pashler (2008) writes that "most current mathematics texts mass practice problems relating to a given topic into one problem set presented immediately following textual presentation of that topic. Our data suggest that—at least for promoting retention—this may be a grievous error" (p. 189). Research further demonstrates that performance is best increased as a result of deliberate, spaced efforts aimed at improvement.
As opposed to massed repetition, deliberate practice consists of tasks that are invented to overcome gaps in understanding, apply what is learned, and deepen understanding and facility with key concepts. These activities are highly structured and designed to improve performance and strengthen understanding. Deliberate practice requires effort and individuals are motivated to practice because practice improves performance (Ericsson et al., 2008). For these reasons, the Pathways are not characterized by long series of similar problems, but rather by carefully chosen questions that guide students to a deeper understanding of concepts.

Instruction supporting the Learning Opportunities: Problem Cycle Routine

The key to realizing the potential of the learning opportunities for students is effective instruction. Modeled after the typical structure of mathematics lessons in Japan (Shimizu, 1999) in which student engagement in rich problems and facilitated discussion of student solutions are key drivers, we have developed an instructional routine, the Problem Cycle, with four phases that faculty can adopt and use strategically to implement lessons in a way that supports the learning opportunities (productive struggle and explicit connections, specifically). Table 1 specifies the purpose and key features of each phase.

TABLE 1
THE PURPOSES AND FEATURES OF THE PHASES OF THE PROBLEM CYCLE ROUTINE

Phase Purpose	Features
Problem Launch: To prepare students for productive struggle - to create a shared understanding of the problem to be worked on, make clear why solving it is important, and stimulate a variety of ways to think about the problem.	• Students are given problems that require explanation as part of their answer and that can be approached in a variety of ways. • Students have a clear understanding of the problem and what they are expected to do. • Students understand why solving the problem is important.
Working the Problem: To engage students in productive struggle with the problem and the concepts and to study students' ways of thinking to prepare for the discussion.	• Students struggle productively with the challenges of the problem. • The instructor recognizes and notes students' ways of thinking in preparation for ensuing discussion.
Discussing the Problem: To make public students' ways of thinking (correct and incorrect), encourage students to learn new ways of thinking by understanding each other, and explicitly connect their thinking to the key concept(s).	• Students present alternative ways of thinking about the problem. • Students have opportunities to analyze mistakes and misconceptions. • Students have opportunities to connect the solution strategies with the key concept(s) and related concept(s). • Students have opportunities to connect the solution to the organizing problem.
Conclusion: To concisely highlight the key concepts drawn from students' thinking, express the concepts with appropriate notation and representations, and explicitly connect the lesson concept(s) with the course organizing concepts.	• Students have an opportunity to see how solving the problem helped them learn the key concept(s). • Students have an opportunity to see how the key concept is related to prior and future concepts. • Instructors clarify formal notation and language to represent key concepts.

To support Pathways instructors to implement the Problem Cycle routine, we have developed a corresponding framework—called the Framework for Improving Teaching (FIT)--that specifies teaching practices to try and those to avoid for each of the phases (see Appendix). For example, for the Discussing the Problem phase, the FIT suggests, among many possible moves to try, that instructors (1) Make explicit the similarities and differences among students' contributions and (2) Explain (or solicit explanations of) how a student's solution strategy related to the key concept of the lesson. It also suggests that instructors avoid calling only on those students known to have the correct answer. These suggestions have been tested by faculty as part of ongoing faculty development activities in the Pathways network.

Productive Persistence

In addition to the foundational learning opportunities described above, the Pathways also focuses on the "non-cognitive" elements of learning. We refer to these non-cognitive elements as Productive Persistence, or the combination of tenacity and good strategies. Research has shown that psychological aspects such as mindsets, "grit" or passion and perseverance for long-term goals, self-control, and engagement are important predictors of academic achievement (see Dweck, Walton, & Cohen, 2014, for a review). However, the critical practical questions are (1) which non-cognitive factors are malleable and (2) how can these factors be affected *reliably, at scale, and by diverse practitioners working in diverse settings.* We sought to answer these questions when we convened researchers and practitioners to develop our Productive Persistence framework, the 5 drivers of which are detailed below.

Students believe that it is possible to learn

When we surveyed our students at the beginning of Statway, we asked them to agree or disagree with the following question: "Being a math person or not is something you really can't change. Some people are good at math and other people aren't." Of the 2174 students sampled in fall 2014, 62 percent agreed with this statement and exhibited what is called a *fixed mindset*. According to Carol Dweck (2006), a fixed mindset refers to the belief that intelligence is a fixed quantity and that no matter how hard you try, you cannot change your intelligence. A fixed mindset strongly relates to the stereotype that being "smart" means being "a natural." For students currently in a fixed mindset, this belief can persist even when they have earned a high grade on a test. Despite being presented with evidence that they can learn, these students may instead attribute their success to luck. The opposite of a fixed mindset is a *growth mindset*, that is a belief that intelligence is malleable. For students in a growth mindset, they value the process of learning rather than just the outcome. For these students, rigorous challenges are not seen as insurmountable obstacles, but rather opportunities to learn and to grow through a combination of effort, good strategies, and asking for help.

Students feel socially tied to peers, faculty, and the course

A vast amount of research on community college student success has found that social ties to peers, faculty, and course of study can affect persistence and engagement (e.g. Steele et al. 2002). We focus primarily on students' *psychological* ties to others—that is, the beliefs and attitudes they have that can limit their ability to feel valued and a part of the learning community. These beliefs and worries about belonging can sap their motivation, even when objective school structures are created to promote belonging. For example, questioning your belonging, what is referred to as "belonging uncertainty," is common and short-lived—except for students who face stigma or negative stereotypes. For these students, belonging uncertainty is heightened, prolonged, and impacts their investment in the class. In a sample of 725 Pathways students, we found significantly higher withdrawal rates for students who reported high belonging uncertainty at the end of the first month (For more details on these analyses, please see Yeager et al, 2013).

Students feel that the material has value

Many students entering developmental mathematics often question the value of mathematics in their daily lives and for their long-term goals. These students may see mathematics more as a roadblock, rather than a stepping-stone on the path to earning their degree and pursuing their goals. One initial step to help students see the value of mathematics is to redesign the curriculum to include relevant material; yet, even with a new curriculum, students must see the work as personally engaging. For example, Hulleman & Harackeiwicz (2009) studied middle-schoolers with low expectations of their success and found that having these students reflect on the short term value, or relevance, of the assignments significantly increased their interest in a topic and, subsequently, their grades.

Students have the Skills, Habits, and Know-How to Succeed in a College Setting

The previous three drivers focused on mindsets and beliefs; however, effective strategies are also an essential aspect of Productive Persistence. Many students in developmental mathematics begin their classes highly motivated to succeed; however, some students get derailed in their pursuits because what is being asked of them is different from what they expected or what they knew how to do. In the data from our learning management system, we have observed a significant negative association between ineffective study strategies and end of course outcomes, even after controlling for background conceptual mathematics knowledge (see Krumm et al, 2015).

Faculty and Colleges Support Students Mindsets and Skills

In order for students to develop these mindsets and strategies, the educational environment needs to be supportive of these shifts. This is important during the first month of the course because many of the students who do not complete a course either withdraw effort or get too far behind (Vaquero & Cebrian, 2013). For that reason, faculty members and researchers co-developed the Starting Strong Package—a combination of 10 instructor-led practical routines and activities that are launched during the first month of class and address the four drivers described above. For example, some routines form a supportive community and establish the norms of collaborative learning, like the Student Group Noticing Routine. In this routine, instructors build a sense of belonging by making students responsible for each other's attendance. The routine consists of three distinct stages. In the first stage, the faculty member puts students into groups and provides time for them to get to know each other outside of the immediate math content using an icebreaker activity (e.g. find 3 non-obvious things that the group has in common). The students also develop a team name and trade their contact information. In the next stage, roughly one week later, groups are responsible for reporting to the faculty who is absent each day. In the final stage, after two weeks of using this routine, groups take responsibility for contacting students who are missing in order to encourage them to attend future classes and give them any materials or information that they missed from class. In classes that actively use this routine, attendance has been strong across the semester (85 percent median attendance rate) and different from past experiences with similar student groups.

The package also includes a brief "growth mindset" reading and writing activity (adapted from Blackwell, Trzesniewski, & Dweck, 2007) and additional practices designed to promote a growth mindset that have significantly decreased students' belief that math intelligence is fixed ($t_{(906)} = -11.854$, $p < .001$, Cohen's D = -0.55, which is a moderate effect size). One powerful way of shifting students' mindsets is to change how learning is discussed. Specifically, focusing on the process of how we learn, the aspect that students can readily control, can positively impact learning. Working together with Carnegie Foundation fellow David Yeager, our network has identified several critical times to start a conversation with a growth mindset phrase:

- When praising students, instead of saying "You're really good at that," a phrase that emphasizes the outcome, you could start a conversation with a phrase that emphasizes the process, like, "You're improving. Your efforts and strategies are really paying off."
- Critical feedback is another high leverage time to begin the conversation with language that promotes a growth mindset. Specifically, Cohen, Steele, and Ross (1999) recommend using phrases that signal that the class has high standards and that you are supportive of your students. For example, you could say: "This class has a high standard to really understand the math AND I wouldn't hold you to it if I didn't believe that together we could get there."
- Additionally, when students are struggling in class, avoid using phrases such as "No one is good at everything, but just try to get through this." Phrases like that suggest that there are just some things that we can't learn and that going through the motions is the most important thing. Instead, start the conversation by reframing the meaning of the students' struggle as part of the process of learning. For example, you could say, "Struggling on this doesn't mean you won't get it. It means you are learning and are making connections that are not yet strong." After starting the conversation with the phrase, we recommend continuing with a discipline-specific discussion of different strategies to approach the problem. Remember, effort is not the only aspect of learning to emphasize; we also need to promote good strategies and asking for help, when needed.

One common misconception about the use of growth mindset language is that it needs to be universally positive; however, as the failure of the self-esteem movement suggests, being positive does not simply translate into better outcomes. Rather, growth mindset language shifts the focus from aspects that students cannot control and that should be seen as irrelevant to learning (e.g. being a "natural" or "smart") to something that they can influence. The goal is for students to see that it is possible for them to learn.

The Pathways target students who are at grave risk of failure in mathematics courses at the community college level—students who have weak K–12 preparation, face language and special education challenges, or fundamentally believe that they are destined to not do well in the subject. The Pathways seek to reverse a pernicious and disheartening cycle of failure by employing materials and teaching approaches that fundamentally put students on a pathway to success. In the Pathways, we have found that instructional practices that are both informed by the Learning Opportunities and address Productive Persistence are key factors in students' success.

See the Appendix for this article on p. 36.

REFERENCES

Bailey, T., Jenkins, D., & Leinbach, T. (2005). *Community college low-income and minority student completion study: Descriptive statistics from the 1992 high school cohort.* New York: Columbia University, Teachers College, Community College Research Center.

Blackwell, L. S., Trzesniewski, K. H., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child Development, 78*(1), 246–263.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.) (1999). *How People Learn: Brain, Mind, Experience, and School.* Washington, DC: National Academy Press.

Cohen, G.L. Steele, C.M., & Ross, L.D. (1999). The mentor's dilemma: Providing critical feedback across the racial divide. *Personality and Social Psychology Bulletin*, *25*, 1302–1318.

Dweck, C.S. (2006). *Mindset*. New York: Random House.

Dweck, C., Walton, G., & Cohen, G. (2014). *Academic Tenacity: Mindsets and Skills that Promote Long-Term Learning*. Bill & Melinda Gates Foundation.

Ericcson, K. A. (2008). Deliberate practice and acquisition of expert performance; A general overview. *Academic Emergency Medicine,15,* 988–994.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Greenwich, CT: Information Age.

Hulleman, C. S., & Harackiewicz, J. M. (2009). Promoting interest and performance in high school science classes. *Science*, *326*, 1410–1412.

Krumm, A. E., D'Angelo, C., Podkul, T., Feng, M., Yamada, H., Beattie, R., Hough, H., & Thorn, C. (2015, 14-15 March). *Practical measures of learning behaviors*. Paper presented at the ACM Conference on Learning @ Scale (L@S'15), Vancouver, BC, Canada.

Pashler, H., Rohrer, D., Cepeda, N. J., & Carpenter, S. K. (2007). Enhancing learning and retarding forgetting: Choices and consequences. *Psychonomic Bulletin & Review*, *14*, 187–193.

Shimizu, Y. (1999). Aspects of mathematics teacher education in Japan: Focusing on teachers' roles. *Journal of Mathematics Teacher Education, 2*(1), 107–116.

Sowers, N. & Yamada, H. (2015). *Pathways Impact Report.* Stanford, CA: Carnegie Foundation for the Advancement of Teaching.

Steele, C. M., Spencer, S. J., & Aronson, J. (2002). Contending with group image: The psychology of stereotype and social identity threat. In M.P. Zanna (Ed.), *Advances in Experimental Social Psychology* (Vol. 34, pp. 379–440). San Diego, CA: Academic Press.

Vaquero, L. M., Cebrian, M. (2013). The "rich club" phenomenon in the classroom. *Nature (Sci Rep)*, *3*, 1174.

Yeager, D., Bryk, A. S., Muhich, J., Hausman, H., & Morales, L. (2013). Practical measurement. Palo Alto, CA: Carnegie Foundation for the Advancement of Teaching.

Appendix

Problem Cycle: Phases, Purposes, Guiding Questions, and Moves* (abridged)

Problem Launch (Purpose): To prepare students for productive struggle—to create a shared understanding of the problem to be worked, make clear why solving it is important, and stimulate a variety of ways to think about the problem.		
Guiding Questions	**Instructional Moves: Do More of These**	**Instructional Moves: Do Less of These**
Launch 1: Are students given a problem to work on that requires explanation as part of their answer and that can be approached in a variety of ways?	• Instructor makes clear that students need to be able to explain their approach and why their solution makes sense (not just how they found it or what it is). • Instructor makes clear that students are expected to take an approach that makes sense to them.	• *Instructor prescribes a particular approach to the problem.*
Launch 2: Do students have a clear understanding of today's problem and what they are expected to do?	• *Instructor devotes time to presenting the problem.*** • Instructor explains background knowledge needed to begin working on the problem. • Instructor provides clear description of the problem goal. • Instructor asks students to restate problem, including what is expected of them. • Instructor asks questions to check understanding of problem, including context relevant to the problem.	• Instructor asks students to begin assignment without prior discussion. • *Instructor provides too much information and reveals a solution strategy.* • Instructor introduces multiple ideas leading to possible student confusion
Launch 3: Do students understand why solving the problem is important?	• Instructor explicitly states the learning goal and explains its significance within the goals of the course. • Instructor discusses questions that illustrate the utility of today's key concept(s).	

Working the Problem (Purpose): To engage students in productive struggle with the problem and the concepts and to study students' ways of thinking to prepare for the discussion. The purpose of this phase is NOT to ensure that all students get the correct answers.		
Guiding Questions	**Instructional Moves: Do More of These**	**Instructional Moves: Do Less of These**
Working 1: Are students struggling productively with the challenges of the problem?	• If students appear stuck, the instructor provides timely hints and examples. • *If students appear stuck, the instructor suggests collaboration with others who are on-track with their thinking.* • If students finish quickly or appear to need challenge, the instructor probes any misconceptions or provides extension question to get them to go deeper into the problem.	• Instructor tells students whether their answers are right or wrong. • *Instructor provides too much information reducing the cognitive demand of the problem (e.g., shows student how to get the answer; asks too many fill-in-the-blank questions).* • Instructor does most of the work required by a task. • If students get stuck for an excessively long time, the Instructor doesn't intervene.

	• Hints are given to keep students struggling only when related to the core issue in the lesson When students are working in groups... • If one (or more) are not actively participating, the instructor asks them to collaborate (i.e., share their thinking and make sense of each others' thinking). • If conversation wanders off task, the instructor redirects students to the problem. • If a student asks a question, the instructor redirects that question to engage the group members. • If a student appears to be dominating the discussion, the instructor intervenes to engage passive students to encourage them to ask clarifying questions.	• If students finish quickly and merely wait for others to catch up, instructor doesn't intervene. When students are working in groups... • *If students' conversations wander off task, Instructor fails to redirect them.* • If an individual dominates the group process, merely showing others how to do the problem, the instructor fails to redirect them.
Working 2: Does the instructor recognize and note students' ways of thinking in preparation for ensuing discussion? (Describes moves that create learning opportunities realized during the discussion.).	• *Instructor observes and studies students' work and student ways of thinking and takes notes.* • *Instructor asks students to prepare their contributions for presentation.* If students are in groups, instructor encourages students to share alternative methods with the rest of the group.	• *Instructor does not actively observe student interactions while waiting for students to finish the task.* Instructor is focused on fielding individual student questions, so unable to note progress being made

Discussing the Problem (Purpose): To make public students' ways of thinking (correct and incorrect), encourage students to learn new ways of thinking by understanding each other, and connect their thinking to the key concept(s).

Guiding Questions	**Instructional Moves: Do More of These**	**Instructional Moves: Do Less of These**
Discussing 1: Do students present alternative ways of thinking about the problem?	• *The instructor calls on students to show a building of ways of thinking toward a main idea.* • Instructor deliberately notes and discusses students' incorrect ways of thinking about the problem when those ways are helpful for developing understanding of concept. • When a student presents his/her work, the instructor follows up with questions that probe the student's thinking (i.e., why answer makes sense, why they think it's true). • Instructor helps all students understand each student's presentation. • *Instructor makes explicit the similarities and differences among students' contributions.* • *Instructor encourages students to ask questions in order to understand their peers' thinking.*	• *Instructor calls only on students who volunteer.* • Instructor calls only on those students known to have the correct answer. • The instructor does nothing more than collect student answers. • The instructor makes little effort to elicit student *thinking* (just fishing for "right answers"). • Instructor treats all responses as equally valuable without regard to the goals of the lesson.

Discussing 2: Do students have an opportunity to analyze mistakes and misconceptions?	• *Instructor maintains a culture in which students feel safe sharing and discussing their incorrect solution strategies.* • *Instructor values incorrect approaches as a learning tool and uses incorrect answers to explore the mathematics.* • Instructor is willing to admit his/her mistakes and uses them to model good mathematical character. • Instructor asks questions that help students understand the reasons why incorrect strategies don't work.	• *Instructor avoids calling on students known to have an incorrect answer, for fear of embarrassing them or for fear of confusing others.* • When a student gives incorrect answer, instructor calls on someone else without responding to the thinking.
Discussing 3: Do students have opportunities to connect the solution strategies with the key concept(s) and related concept(s)?	• Instructor asks students to explain connections between concepts. • Instructor asks students to use key concept(s) to explain why their solution strategies work. • Instructor asks questions that help students connect the key concept(s) with related concepts? • *Instructor draws attention to the different contributions made by different ways of thinking to the mathematical point of the lesson.*	• *Instructor ends work on the problem as soon as the answer is made public.* • *Instructor responds to all ways of thinking in the same way without drawing attention to the connection each has to the mathematical point.*
Discussing 4: Do students have opportunities to connect the solution to the organizing problem?	• Instructor asks students to reflect on the reasonableness of the solution with respect to the scenario. • Instructor reflects on how doing the mathematical thinking adds to the knowledge about the scenario • Instructor pays attention to how variables, graph labels, etc. are used to make connections to the scenarios	• Instructor does not relate the solution back to context of the problem.

Conclusion (Purpose): To concisely highlight the key concepts drawn from students' thinking, express the concepts with appropriate notation and representations, and connect the lesson concept(s) with the course organizing concepts.		
Guiding Questions	**Instructional Moves: Do More of These**	**Instructional Moves: Do Less of These**
Conclusion 1: Do students have an opportunity to see how solving the problem helped them learn the key concept(s)?	• The instructor connects student work to the key mathematical concept(s) by incorporating several quotes that highlight the progression of student thinking that developed in the lesson.	• Instructor ends work on the problem as soon as the answer is stated. • The concept is recited by the teacher, but with little connection to student work and discussion.
Conclusion 2: Do students have an opportunity to see how the key concept is related to prior and future concepts?	• *Instructor provides a coherent statement of the key concept(s) of the lesson.* • *Instructor situates the key concept(s) of this lesson within the mathematical trajectory for the course.*	• The summary is out of focus or mathematically incorrect.

Conclusion 3: Does the instructor clarify formal notation and language to represent key concepts?	• Instructor connects notation and language to concepts already discussed. • Instructor treats new notation and language as useful and efficient ways to represent familiar procedures or concepts.	• Instructor uses language in an imprecise or incorrect way.

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* Moves are continuously tested and updated in the Pathways network professional development activities.
** Moves in *italics* are good candidates to focus on initially.

Ann R. Edwards is the director of Learning and Teaching at the Carnegie Foundation for the Advancement of Teaching in Stanford, California; Rachel L. Beattie is the director of Productive Persistence, also at the Carnegie Foundation.

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